Problem 1: Induced Graphs. Answer questions about the graph G = (V,E) displayed below.

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1. Let U = {A, B}. Draw G[U]

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1. Let W = {A, C, G, F}. Draw G[W]

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1. Let Y = {A, B, D, E}. Draw G[Y]

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1. Consider the following subgraph H of G. Is there a subset X of the vertex set V so that H = G[X]? Explain

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🡪 No, there can’t be. G[X] will be:

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1. Find a way to partition the vertex set V into two subsets V1, V2 so that each of the induced graphs G[V1] and G[V2] is connected and G = G[V1] U G[V2].

🡪 V1={D, E, I} and V2={A,B,C,F,G,H}

Problem 3:

1. Suppose G = (V, E) is a connected simple graph. Suppose V1, V2, . . ., Vk are disjoint subsets of V and that V1 U V2 U . . . U Vk = V. Show that there is an edge (x,y) in E such that for some i < j, x is in Vi and y is in Vj
2. In class it was shown that a graph G = (V, E) is connected whenever the following is true, (\*) m > C(n-1, 2) where n is the number of vertices and m is the number of edges. Is the following true or false? Every connected graph satisfies the inequality (\*). Prove your answer

🡪 Below graph is connected with m = 5, n = 6 but m < C(n – 1, 2)

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1. Suppose G is a graph with two vertices. What is the minimum number of edges it must have in order to be a connected graph? Suppose instead G has three vertices; what is the minimum number of edges it must have in order to be connected? Fill in the blank with a reasonable conjecture:

🡪 If G has n vertices, G must have at least **n - 1** edges in order to be connected.